NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF AGRICULTURE AND NATURAL RESOURCE SCIENCES
DEPARTMENT OF AGRICULTURAL SCIENCES AND AGRIBUSINESS

QUALIFICATION : BACHELOR OF SCIENCE IN AGRICULTURE			
QUALIFICATION CODE:	07BAGA	**LEVEL:**	7
COURSE CODE:	MTA611S	**COURSE NAME:**	Mathematics for Agribusiness
SESSION:	June 2023	**PAPER:**	Theory
DURATION:	3 Hours	**MARKS:**	100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
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MODERATOR(S)	Mr. Teofilus Shiimi

INSTRUCTIONS

1. ANSWER ALL questions.
2. Write clearly and neatly.
3. Number the answers clearly & correctly.

PERMISSIBLE MATERIALS

1. All written work MUST be done in blue or black ink.
2. Calculators allowed.
3. The **LAST PAGE** has **FORMULA.**
4. No books, notes and other additional aids are allowed.

THIS QUESTION PAPER CONSISTS OF 6 PAGES (including this front page).

QUESTION ONE **[MARKS]**

a. Consider a function, $f(x) = x^2 - 4x - 5$. Find the range when the domain is one and the domain when the range is zero. (4)

b. Use interval notation to express the domain of the function: (4)

$$g(x) = \frac{2x - 1}{x^2 - 9}$$

c. Suppose you know that an agribusiness's production can be approximated using a univariate quadratic function with a maxima and roots at $x = -10$ and $x = 20$. Based on this information answer the following questions below.

i. Derive the algebraic equation for the production function. (2)

ii. Compute the production function's y-intercept. (2)

iii. Compute the range and domain value at the maximum point. (3)

iv. Sketch a well labelled graph to represent the production function. On your graph show the roots, y-intercept, and maxima. (5)

d. A vendor's total monthly revenue is from the sale of x bags potatoes is represented by a function:

$$r = 150x$$

Furthermore, the vendor's total month costs are given by $c = 100x + 3500$. Compute, how many bags of potatoes must the vendor sale to break even? (*Hint: break even means revenue is equal to cost*). (5)

TOTAL MARKS **[25]**

QUESTION TWO	[MARKS]

a. Use the Newton's Difference Quotient (or first principle of differentiation) to find the first derivative of the function: (6)

$$g(x) = x^2 - 4x - 5$$

To obtain full marks, show all the critical steps in your answer.

b. Find:

i. $\lim_{x \to 0} \frac{(2+x)^2 - 4}{x}$ (4)

ii. $\lim_{k \to 6} \frac{\sqrt{k-2} - 2}{k - 6}$ (6)

c. Find the equation of a straight-line that is tangent to the curve:

$$y = \ln(x^2 - 2x + 24)$$

at $x = 0$. (9)

TOTAL MARKS **[25]**

QUESTION THREE **[MARKS]**

a. Consider the functions, $f(x) = (3x^4 - 5)^6$ and $g(x) = \log_8 x^4$. Find:

i. $f'(x)$ (3)

ii. $g'(x)$ (4)

b. Find z_x, z_y and z_{yx}, given the function:

$$z = 3e^{2x}y^2$$ (6)

c. Find the critical points of the function below and test whether it is at a relative maximum, relative minimum, inflection point, or saddle point. Show all your calculations. (12)

$$z = 3x^3 - 5y^2 - 225x + 70y + 23$$

TOTAL MARKS **[25]**

QUESTION FOUR [MARKS]

a. Find:

i. $\int_0^1 (3x^2 - x - 2)\, dx$ (3)

ii. $\int x^2(x^3 + 2)dx$ (5)

b. Suppose an agribusiness's marginal cost function of wheat production is represented by:

$$MC = \frac{dc}{dq} = 250 + 30q + 9q^2$$ (7)

where MC is the marginal cost, c is the total cost, and q is the units of output. Find the cost of producing 10 units of output assuming a fixed cost of N$10,000.

c. To produce 70 tonnes of wheat, an agribusiness wishes to distribute production between its two farms, farm 1 and farm 2. The total cost of wheat production, c, is given by the function:

$$c = 4q_1^2 + 2q_1q_2 + 5q_2^2 + 1000$$ (10)

where q_1 and q_2 are tonnes of wheat produced at farm 1 and farm 2, respectively. How should the agribusiness distributed to production between the two farms to minimize costs? Furthermore, compute and interpret lambda (λ).

TOTAL MARKS **[25]**

THE END

FORMULA

Basic Derivative Rules

Constant Rule: $\frac{d}{dx}(c) = 0$

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Sum Rule: $\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$

Difference Rule: $\frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$

Product Rule: $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x)+g(x)f'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$

Chain Rule: $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$

Derivative Rules for Exponential Functions

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(e^{g(x)}) = e^{g(x)}g'(x)$$

$$\frac{d}{dx}(a^{g(x)}) = \ln(a)a^{g(x)}g'(x)$$

Derivative Rules for Logarithmic Functions

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, x > 0$$

$$\frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x\ln a}, x > 0$$

$$\frac{d}{dx}(\log_a g(x)) = \frac{g'(x)}{g(x)\ln a}$$

Basic Integration Rules

1. $\int a\,dx = ax + C$
2. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{1}{x}dx = \ln|x| + C$
4. $\int e^x\,dx = e^x + C$
5. $\int a^x dx = \frac{a^x}{\ln a} + C$
6. $\int \ln x\,dx = x\ln x - x + C$

Integration by Substitution

The following are the 5 steps for using the integration by substitution metthod:

- Step 1: Choose a new variable ***u***
- Step 2: Determine the value ***dx***
- Step 3: Make the substitution
- Step 4: Integrate resulting integral
- Step 5: Return to the initial variable ***x***

Integration by Parts

The formula for the method of integration by parts is:

$$\int u dv = u \cdot v - \int v du$$

There are three steps how to use this formula:

- Step 1: identify ***u*** and ***dv***
- Step 2: compute ***u*** and ***du***
- Step 3: Use the integration by parts formula

Unconstrained optimization: Univariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

- Step 1: Find the critical value(s), such that:

  $$f'(a) = 0$$

- Step 2: Evaluate for relative maxima or minima
  - If $f''(a) > 0 \rightarrow$ minima
  - If $f''(a) > 0 \rightarrow$ maxima

Unconstrained optimization: Multivariate functions

The following are the steps for finding a solution to an unconstrained optimization problem:

Condition	*Minimum*	*Maximum*
FOCs or necessary conditions SOCs or sufficient conditions	$f_1 = f_2 = 0$ $f_{11} > 0$, $f_{22} > 0$, and $f_{11} \cdot f_{22} > (f_{12})^2$	$f_1 = f_2 = 0$ $f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} > (f_{12})^2$
	Inflection point	
	$f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$ or $f_{11} < 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$	
	Saddle point	
	$f_{11} > 0$, $f_{22} < 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$, or $f_{11} < 0$, $f_{22} > 0$, and $f_{11} \cdot f_{22} < (f_{12})^2$	
	Inconclusive	
	$f_{11} \cdot f_{22} = (f_{12})^2$	

Constrained Optimization

The following are the steps for finding a solution to a constrained optimization problem using the Langrage technique:

- Step 1: Set up the Langrage equation
- Step 2: Derive the First Order Equations
- Step 3: Solve the First Order Equations
- Step 4: Estimate the Langrage Multiplier